The DICE is loaded: 3 papers that change the rules of Nordhaus' model

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Abstract

This note details the basic equations on which DICE, the most standard IAM, is based, and explore how changing them can imply highly different results. Relying on existing research, we especially highlight how CBA-IAM can recommand strong mitigation policies.

1 The DICE model

The Dynamic Integrated Climate-Economy (DICE) model is a cornerstone in climate economics. It has been developed by William Nordhaus for the first time in 1992 [30], and has been lastly updated in 2023 [5][1]. DICE adopts a top-down approach to model the macroeconomic impact of climate change and the associated policies from a long-term perspective. As a Ramsey-Cass-Koopmans model of economic growth, it seeks to find the optimal (thus normative) transition path built on a cost-benefits analysis relying on deterministic, dynamic and recursive computations. Aggregated at the world level, it doesn't leave place for distributional issues, whether of income or climate-related cost. The hypotheses underpinning DICE are very standard: a unisectoral model without any explicit energy sector and a supply-based output. The production is defined as a Cobb-Douglas function, such that capital K and labor L are substitutable.

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}, \quad \alpha \in]0;1[\tag{1}$$

Every individual is considered working (Population = Labor). Labor and Total Factor Productivity (TFP, noted A) are exogenous and their growth rates decline. Capital is endogenously calculated through its conventional law of motion and a neoclassical closure (saving-investment equality assumption) happens at each time period.

$$K_{t+1} = (1-\delta)K_t + I_t \tag{2}$$

Individuals are defined under a single (so-called) representative rational agent. DICE sets as an optimal control method the maximisation of intertemporal utility with a regular discounted total utilititarianism criterion. The utility function is a standard Constant Relative Risk Aversion (CRRA) function depending only on consumption per capita. Denoting W the intertemporal welfare function being maximised, we have:

$$W = \sum_{t=1}^{T} \frac{1}{(1+\rho)^t} L_t \frac{c_t^{1-\eta}}{1-\eta} \tag{3}$$

As a Cost-Benefits Analysis Integrated Assessment Model (CBA-IAM), DICE represents interactions between climate change and economic modules. From carbon emissions, a carbon cycle model generates atmospheric carbon concentration. This concentration is translated into a temperature change which impacts the economy (through a damage function). Mitigation can be implemented at a certain cost to reduce emissions, depending on global aggregate emissions and emissions abatement. There is neither technical change nor adjustment cost, and mitigation cannot bear a potential economic benefit. Damages and mitigation can affect welfare, growth, and the emissions that are generated from economic production.

In DICE-2013R, CO_2 emissions from production (through the carbon intensity of out-

[1] For the sake of consistency with the presented papers, the following technical aspects of DICE rely on the 2013 version [29].

put σ_t and the mitigation rate μ_t) and from land-use change are taken into account. The carbon cycle is modeled through three reservoirs: the atmosphere (M), the biosphere and upper layers of the ocean (M^u) and the deep ocean (M^l). The M^i variables measure the carbon concentration, and carbon circulation is allowed between reservoirs. The global mean surface temperature increase T_t depends on the radiative forcing RF_t (given by the Arrhenius formula), the equilibrium climate sensitivity S (*i.e.* the long-term temperature increase after having reached a doubling of the atmospheric CO_2 concentration), the speed of adjustment to the new temperature (ξ_1), Planck's feedback ($\xi_2 T_{t-1}$) and ocean's temperature (T^l.)

$$T_t = T_{t-1} + \xi_1 \Big[\mathrm{RF}_t - \xi_2 T_{t-1} - \xi_3 \left(T_{t-1} - T^l_{t-1} \right) \Big], \quad \xi_2 = \frac{F_{2\times CO_2}}{S} \tag{4}$$

The damage function is assumed to be a polynomial of degree 2 in temperature increase, such that:

$$\Omega_t = \psi_1 T_t + \psi_2 T_t^2, \quad \psi_1 \geq 0, \psi_2 > 0 \tag{5}$$

The temperature leading the reduced-form damage function seeks to represent a wider set of climatic variables (sea-level rise, precipitations, etc.).

The abatement costs are represented with a monotonous rising function, dependent on the emission control rate μ_t, which translates as the ratio between the marginal costs of abatement over the price of the backstop technology.

$$\Gamma_t = a_t \mu_t^\theta, \quad a_t, \theta > 0 \tag{6}$$

DICE supposes that a_t declines exogenously with time, which implies that emission reduction cost is independent of previous emission levels.

Eventually, the output net of climatic damages and abatement costs is written:

$$Q_t = \frac{1 - \Gamma_t}{1 + \Omega_t} Y_t \tag{7}$$

As an optimisation model, DICE allows to compute the Social Cost of Carbon (SCC), defined as the present value of the damage stream resulting from a marginal unit of carbon emissions:

$$SCC_t = \frac{\partial W}{\partial E_t} \left(\frac{\partial W}{\partial C_t} \right)^{-1} \tag{8}$$

The SCC is used to determine the optimal carbon price and inform the optimal mitigation policy: for instance, the US Environmental Protection Agency uses the DICE model to set their own value of the SCC [3].

Although it allows potential decarbonisation levers to be explicitly represented through stylized facts, much criticism arised during the last decades, mostly because, under Nordhaus' assumptions, DICE's lack of realism leads to very moderate recommandations about the economic benefits of the transition, undermining ambitious climate policies.

In his last paper, the optimal cost-benefit policy corresponds to a Social Cost of Carbon worth only 50 2019USD/tCO_2 for 2020, leading to a 2.6 °C increase in 2100 [5], while a contemporary study gives a value twenty-fold higher, of 1,056 2024USD/tCO_2 [8]. Beyond this huge gap, an overly simplistic representation of the climate-economy relationship is at work in DICE, to which can be added the lack of moral and ethical considerations, notably on the distribution of costs and benefits of climate policies. Many authors have thus suggested to include new mechanisms (differentiated consumption to obtain relative price changes [42], stochasticity and uncertainty [20, 25], inequalities [15]) or to challenge some hypotheses over parameters (the most famous being the value of the discount rate ρ discuted in the Stern Review [40]) in order to demonstrate how DICE could provide optimal trajectories that are actually compatible with the Paris Agreement objectives. In the following, we explore how the papers of Dietz and Stern (2015), Moore and Diaz (2015) and Grubb *et al.* (2020) [19, 21, 27] question the DICE model and reveal some of its weaknesses.

2 Extending DICE

A general way of posing their research question would be: "How relaxing unrealistic assumptions can affect DICE's optimal climate policy?". While Dietz and Stern [19] and Moore and Diaz [27] focus on better describing the effects of temperature increase on economic growth, Grubb et al. [21] rather emphasize the importance of taking into account the dynamic interdependencies that underpins the trajectories of the emissions abatement costs, for instance by considering the combination of inertia and technology learning. Dietz and Stern especially shed a light on the interaction of a new functional form for the damage function, tipping points, uncertainties and an endogenized growth mechanism, while Moore and Diaz explored the combination of regionalisation and endogenous growth.

To better account for welfare loss, Dietz and Stern built on the wise work of Martin Weitzman (1942 - 2019) [48] and defined a reactive damage function anchored in order to take into account tipping points (*eg.* at + 12 °C, the output is reduced by 99%).

$$\Omega_t^{\text{D\&S}} = \psi_1 T_t + \psi_2 T_t^2 + \psi_3 T_t^{6.754} \tag{9}$$

$$D_t^{\text{D\&S}} = 1 - \frac{1}{1+\Omega_t^{\text{D\&S}}} \tag{10}$$

Weitzman insisted on the structural uncertainties upon which CBA-IAM are based, whether they are about the tail fatness of the probability density function of the diswelfare caused by climate change, or about the potential brutal changes that cannot be captured by models that are based on smooth functions. His views were quite aligned with Pindyck's [35] about the (too) wide range of possible policy recommandations relying on unkownable assumptions. Dietz and Stern then chose to leave DICE's deterministic framework by replacing the fixed value of equilibrium climate sensitivity ($S = 3$ °C) by a probability density function with a fat right-hand tail (*i.e* positively skewed, with a significantly higher probability of extreme events compared to a normal distribution), such that its mean value μ_S and its standard deviation σ_S are $\{\mu_S, \sigma_S\} = \{2.9, 1.4\}$ °C. With respect to the equation (4), S is now replaced by $f(S)$ such that:

$$f(S) = \frac{a(\frac{S}{b})^{a-1}}{b(1+(\frac{S}{b})^a)} \tag{11}$$

Dietz and Stern also introduced endogenous growth à la Romer, *i.e.* accumulation of knowledge (through learning-by-doing) that spillovers over all firms, with damages, throughout two versions of the model.

The first one adds explicit damages on capital stocks, and knowledge spillovers are incorporated *via* capital into the production function (with a productivity factor K^β).

$$Y_t^{\text{D\&S-1}} = A_t K_t^{\alpha+\beta} L_t^{1-\alpha}, \quad \alpha, \beta \in]0;1[\tag{12}$$

$$K_{t+1}^{\text{D\&S-1}} = (1-\delta^k)(1-D_t^k)K_t + I_t \tag{13}$$

$$Q_t^{\text{D\&S-1}} = (1-\Gamma_t)(1-D_t^Y)Y_t^{\text{D\&S-1}} \tag{14}$$

The second one instead endogenize TFP (with a slower depreciation than capital), such that the law of motion for capital is the same as in DICE 2013 (2), but the TFP and the production function are:

$$\tilde{A}_{t+1} = (1-\delta^A)(1-D_t^A)\tilde{A}_t + \gamma_1 I_t^{\gamma_2} \tag{15}$$

$$Y_t^{\text{D\&S-2}} = \tilde{A}_t K_t^{\alpha} L_t^{1-\alpha}, \quad \alpha \in]0;1[\tag{16}$$

$$Q_t^{\text{D\&S-2}} = (1-\Gamma_t)(1-D_t^Y)Y_t^{\text{D\&S-2}} \tag{17}$$

In this model, the term $\gamma_1 I_t^{\gamma_2}$ allows a TFP growth through knowledge spillovers coming from capital investment. Contrary to the first version, the knowledge thus does not depreciate as fast as the capital stock and its loss is captured by the damages on TFP.

Let us point out that damages are partitioned between output Y and $i, i \in \{A, K\}$ such that:

$$D_t^i = \lambda^i D_t, \quad \lambda^i \in [0;1] \tag{18}$$

$$D_t^Y = 1 - \frac{1-D_t}{1-D_t^i} \tag{19}$$

Having endogenous drivers of growth that can be affected by climate change allows to depict what is called a "growth-effect", a dynamic relationship between GDP growth rates and damages such that the output possibilities are permanently reduced. It traduces more realistic assumptions about climate impacts, *eg.* on capital stock (such as infrastructure or their productivity) from extreme events, on TFP because past investments are not as productive as they would have been in a cooler climate, or on learning-by-searching because of the allocation of resources to adapt to climate change instead of investing in R&D.

Moore and Diaz also represent this growth effect by damages over TFP growth or capital depreciation. Rather than an explicit quadratic function, they use temperature shocks which a persistency that exponentially decays over time, as an adaptation mechanism. The temperature increase T_t is then replaced by an effective temperature ET_t:

$$ET_t = \sum_{i=1850}^{t} (T_i - T_{i-1}) e^{-a(t-i)} \tag{20}$$

They model a differentiated effect on poor and rich countries (on growth rates and economic output), considering that the former will suffer more from warming. They distinguish two mechanisms for an explanation, that have opposite implications. The first one justifies that the damages are a function of the temperature, because poorer countries tend to be hotter than richer ones, therefore a warming might lead to more frequent damaging temperature. The second one is a resilience mechanism, because poorer countries rely more on climate-exposed sectors (*eg.* agriculture). The damages are then a function of GDP per capita.

TFP growth endogenization is similar to that of Dietz and Stern. For a region j and an exogenous annual TFP growth rate $r_{\mathrm{TFP}_{j,t}}$:

$$A^{\mathrm{M\&D}}_{j,t} = (1 + r_{\mathrm{TFP}_{j,t}} - \tilde{\gamma_0}_j ET_t)^{\Delta t} A_{j,t-1} \tag{21}$$

with $\tilde{\gamma_0}_j$ the sensitivity of TFP growth to either temperature or resilience mechanism.

When capital depreciation is endogenized, it is based on values for total factor productivity, labor, investment, baseline growth rate, temperature, and the previous year's output and capital stock. Its value derives from the equality between the net output function written as $Q^{\mathrm{M\&D}}_{j,t} = A_{j,t}\left((1-\tilde{\delta}_j)K_{j,t-1} + I_{j,t-1}\right)^{\alpha} L^{1-\alpha}_{j,t}$ and its expression implying its growth rate, $Q^{\mathrm{M\&D}}_{j,t} = Y_{j,t-1}(1 + r_{\mathrm{g}_{j,t}} - \gamma_{0j} ET_t)^{\Delta t}$, such that:

$$\tilde{\delta}_j = 1 + \frac{1}{K_{j,t-1}}\left[I_{j,t-1} - \left(\frac{Y_{j,t-1}(1 + r_{\mathrm{g}_{j,t}} - \gamma_{0j} ET_t)^{\Delta t}}{A_{j,t} L^{1-\alpha}_{j,t}}\right)^{\frac{1}{\alpha}} \right] \tag{22}$$

It can been shown that a concave quadratic function fits the relationship between temperature and depreciation.

Leaving to others the damage question, Grubb *et al.* highlight the poor representation of abatement costs in DICE compared to the last empirical evidence. To implement dynamic realism, they argue for the importance of induced innovation, inertia and path-dependency as drivers of the technical change of energy systems and their associated costs. Nordhaus did try to include induced innovation in the R&DICE model but found that it would led to more emissions than exogenous innovation with substitution to renewables [31].

The authors challenge this result by reminding that innovation has both an exogenous part (through public R&D and spillover) and an endogenous one (induced by prices, market deployment or demand). They also insist on the major role of path-dependency and inertia in the adoption of low-carbon technologies, which if not taken into account, can lead to a lock-in in high-carbon infrastructures (and more broadly a technological lock-in during the transition. For example, replacing coal with gas will ask heavy investment in new infrastructures and R&D: society thus have to be sure of the direction they give to the transition [2]).

Indeed, the present situation highly depends on past choices through locking mechanisms that can come from technical factors (such as increased performance by spillovers, learning-by-using and learning-by-doing, network externalities or technological interrelatedness), but also behavioral (*eg.* self-reinforcing habits, social norms or sunk-costs fallacy) and institutional [39]. Moreover, because the lifetime of capital stock is non null, society inherits a given set of technologies, industries, institutions and social norms that might maintain the *status quo* through the mechanisms evoked beforehand if no directed technical change is implemented to break the inertia [1].

To reflect a transitional element, Grubb et al. [21] suggests considering the rate of change of abatement $\dot{\mu}_t$ and a characteristic transition timescale $\hat{t}$ in the abatement cost. The function is then composed of an inertial term (how much efforts are needed to switch from path-dependency) and a rigid term (how much are absolute abatement costs) whose proportion is determined by a pliability parameter p. No pliability corresponds to a case without inertia whereas full pliability implies total path-dependency.

$$\Gamma_t^{\text{Grubb}} = a_t \left((1-p)\mu_t^\theta + p\frac{\hat{t}^\theta}{\theta+1}\dot{\mu_t}^\theta \right) \quad (23)$$

Through their work, each of the authors suggest an extension of the standard DICE framework: temperature (4) is challenged by Dietz and Stern (11) and reinterpreted by Moore and Diaz (20), damages (5) are revised (9) and integrated in the growth mechanism (12), (15), (21), (22), and abatement costs (6) are redefined to include inertia and path-dependency (23). Now that a new framework has been established, the need arises to put numbers in it, which means making decision on the data to use to calibrate the parameters and the uncertainties that come with it.

3 Dealing with modeling: choices and uncertainties

Parameters are estimated more or less roughly depending on data availability, econometrics tool, ethical consideration, modeling choices and true knowledge of the world. In DICE, the choices made seem to converge toward a poor estimate of the damages and how they matter for future generations.

First, the damage function's parameters ψ have been calibrated on Tol's (2009) estimates [44], multiplied by 1.25 to virtually account for non-monetized impacts. It is worth noting that Tol's study has been found to be deeply wrong: he showed that a 2.5 °C increase in temperature would lead to a loss of 0.7% in GDP, then published a correction five years later [45] which also happened to be deeply criticized (see for instance Bob Ward's response on the Journal of Economic Perspectives' website).

Then, the aversion to intertemporal inequality is quite low, $\eta = 1.45$, and the pure rate of time preference quite high: $\rho = 1.5\%$/year (which translates as follow: the value of a life in 50 years equals 50% of what it is worth today). It shall be highlighted that Weitzman chose prescriptively to set $\rho = 0$, following Ramsey, Pigou or Solow, and that Stern in its review [40] uses $\rho = 0.1\%$ to account for a small probability of human extinction. In the presented papers however, the authors do not challenge the discount rate for the sake of comparison.

Many econometric studies intend to assess climate change impacts on economic growth and the economy, whether on productive sectors [10, 14] such as agriculture [38] or through a loss of labor productivity due to hotter temperature, or on welfare loss (*eg.* health degradation) [16]. However, they often rely on a +1 °C increase in temperature (or a normalized increase of 1 °C), studying short-run impacts on GDP. Because econometrics can only look in the past, and physics cannot provide a perfect forecast of climate damages and the related GDP growth rate sensitivity, the damage functional form and its parametrisation necessarily lie in radical uncertainty: only "quasi" data-points can be used for calibration.

From then, different modeling choices are made. Moore and Diaz chose to calibrate their regional growth-rate sensitivity to temperature $\tilde{\gamma}_{0j}$ on Dell et al. [14], from time lags and regression coefficient on temperature over GDP growth (and corrected by the feedback caused by endogenous capital calculation). They had to assume a zero long-run effect through an "optimistic adaptation" hypothesis: they set quite arbitrarily the rate of adaptation a at 10%/year such that 95% of the impact of a temperature shock is lost after 30 years (20). To complete their exploration, they perform sensitivity analyses on the adaptation rate and the equilibrium climate sensitivity to assess the robustness of their results.

On the contrary, Dietz and Stern follow a normative approach. Whereas in DICE, +18 °C increase is needed in order to reach a loss of 50% of the output, they parametrize ψ_i such that 50% of the GDP remains either at +6 °C or +4 °C. They still use empirical studies to define the λ^i parameters in the damages share (18), with $\lambda^k = 0.3$ and $\lambda^A = 0.05$, respectively based on Nordhaus and Boyer [32] and Moyer et al. [28], though high uncertainties remain on these values. To explicitly compare their extended version to Nordhaus' one, they use most of the same parameters (labor, abatement cost, savings rate, etc.) and calibrate the spillover function $\gamma_1 I_t^{\gamma_2}$ (15) such that the output without climate change replicates DICE's results.

Finally, knowing the high uncertainty falling on the equilibrium climate sensitivity, Dietz and Stern combine a probability distribution calibrated on the IPCC AR5 [43], using a log-logistic form (11) as a "middle of the road" assumption. Even tough they go beyond the deterministic framework, they do not implement learning about climate sensitivity from observation, it is only taken as given before the first period starts. Still, they perform a complete investigation by performing a sensitivity analysis on the ECS, conscious of the substantial uncertainty that lies on this parameter.

As for Grubb *et al.*, they have the advantage to rely on a wide range of recent empirical studies to calibrate their abatement cost function (23). For instance, they refer to Bashmakov et al. [6] for insights on the necessary time to adapt the energy system in OECD countries after the 1970s oil shocks (25-33 years), they identify as a major factor of inertia the 40-year lifetime of coal plants, and highligh that carbon lock-in for transport infrastructure can last for centuries and that market penetration of new technologies takes time, with a wide range of time-scales depending on technologies, sectors, socio-economic factors, etc. Overall, they deduce that the transition time-scales are at least 20-40 years for the bulk emitting systems and choose in consequence to set $\hat{t} = 30$ years. For reproducibility, they use the same parameter θ as in DICE, and they explore the sensitivity of the results to the pliability parameter p, with $p \in \{0; 0.5; 1\}$. They justify these values by the empirical literature, which suggests that p should be between 0.5 and 1 and use $p = 0$ to reproduce DICE's results.

4 Results

The computed scenarios start as soon as 2015 and end between 2100 and 2245. They often integrate a baseline scenario (without any mitigation policy) and an optimal control scenario of reference, corresponding to DICE's optimal policy. Then, they provide new trajectories to explore their suggested improvements.

Nearly all scenarios show a striking decrease in CO_2 emissions compared to the reference scenario allowing some of them to fit the Paris Agreement objectives. Because of the many uncertainties around the model, it must be given more importance to the ordinality of the results rather than their absolute values.

Still, we can denote than simulated DICE-like baselines lead to a temperature increase between 3.5 and 4.5 °C by 2100 and that the optimal scenario of reference (mitigation policies with Nordhaus' parameters) allows for a $\sim$ 3 °C warming. Using the described mechanism, Dietz and Stern and Moore and Diaz succeed in showing how higher damages and growth-effect strongly impede the economy. More precisely, cases where 50% of global output is lost at +4 °C or +6 °C (and/or combined with a high S value) reverse the consumption per capita curve, traducing a collapse in living standards because of damages on productivity growth. The scenario with Weitzman damages and a high climate sensitivity value leads to an optimal net-zero emissions by 2055 in the TFP model and 2065 in the K model, and a carbon price over 100 2012USD/tCO_2 in 2025[2]. It shall be noted than when warming is fast, the growth effect matters less: as it is a compounded effect over time (a cumulative sum of small impacts), it becomes more similar to a level-effect if instantaneous damages are high.

Switching the climate sensitivity to a probability distribution function narrow the allowed emissions, as a risk effect mechanism.

When regional heterogeneity is included, some of the previous results are altered. While the growth effect is still present, it is less pronounced in rich regions, which are more resilient to temperature shocks. However, poor regions assists to a 40% reduction in GDP per capita by 2100. Considering a temperature mechanism, the optimal mitigation implies reaching net zero no later than 2070, a result robust to changes in the adaptation rate a (from 0% to 20%) ; but even so, they still loose 20% in GDP per capita by 2100 compared to a scenario without climate change.

Nonetheless, the major result of Moore and Diaz reside in the non-robustness of the growth effect to the mechanisms driving growth-rate impacts. While the temperature mechanism leads to reach the Paris Agreement with a high Social Cost of Carbon, the simulation of a re-

[2] N.B. : the 2024 EU ETS price fluctuate between 60 and 80 eur.2024/tCO_2

silience mechanism results in a relaxation of mitigation in favor of economic growth. Indeed, it allows to reduce poor countries' sensitivity to exposed sectors by developing quickly, which leads to a rise in emissions that can imply a +6 °C warming by 2150 (5 °C by 2100). Therefore, the true explanation of poor countries' vulnerability to climate change is crucial to decide whether to prioritize decarbonization or resilience through growth.

In a similar way, Grubb *et al.* reveal how properly representing temporal interdependence influences the policy recommendations of IAMs. Depending on pliability, the net-zero target can be reach after 2120 (standard DICE recommandation) or as early as 2070 (if there is a full pliability in the energy system). More than quantitative results, they show how different can be the temporal distribution of cost whether inertia of systems combined with learning is taken into account or not. With substantial pliability, starting with high investment sustained at their initial level for few decades drive the low-carbon innovation, overcome inertia and break the path-dependency upon fossil fuels. Then, costs decline until reaching quickly a low level, along with a decrease in climate damages compared to a scenario without inertia (where costs keep increasing). The authors underline the coherent results they draw compared to more complex hybrid models such as IMACLIM-R [13].

Nevertheless, the authors insist on the fact that the models, especially top-down ones, underperform in capturing the complexity of the energy transition and its mechanisms (such as induced innovation) and plead for a diversification of reference paths as there is no unique, least-cost pathway for the global energy system, because of the diversity of dynamic linkages across sectors and technologies. More than a single global carbon price should be implemented to induce the energy transition, and it should especially be complemented by targeted instruments to induce mitigation in long-lived infrastructure.

5 Discussion

In the IAM community is admitted the inevitable trade-off between interpretation of transparent insights and complexity that reflects reality. DICE stands in the first category, and its simplicity allows to play with the mechanisms that underpin the climate-economy relationship. Nonetheless, it necessarily makes it very sensitive to many hypotheses (almost all of them actually). Welfare function, aversion to intertemporal inequality, amount of damages, representation of costs, time discounting, the framework of utilititarianism (for a reflection and proposal for other criteria, see for instance [4, 9, 18, 49]), etc.

One can argue that a more rigorous accounting of the effects of climate change on the economy could justify more stringent mitigation policies, as we have seen in our three papers. More recent econometric studies could be used to calibrate the damage function [23, 24] ; however, they might face missing data or difficulties to capture the full range of impacts. Other works try to develop the damage functional form: for instance, Da Costa (2024) noted that the shape of DICE's damage function crushed high values and suggested a damage function based on a growth-effect mechanism [12], based on Bilal and Känzig recent work [8].

New extensions try to introduce stochasticity [46], tipping points [11] or spatial structure [17]. Recent papers on uncertainty involve the use of deep learning [37] or statistical physics [47] to better understand the mechanisms in place.

However, some persistent uncertainties and dilemma cannot be removed. For instance, Stern argues that from a moral point-of-vue, pure-time discounting is essentially discrimination by date of birth. On an other hand, no matter our efforts in quantifying the damages, radical uncertainty remains at the same time on the climate system and on the economy, such that we could cross physical tipping points by 4 °C above pre-industrial levels [26], but also socio-economic tipping points (conflicts, migration, etc. [7, 41]).

Some thing we are sure of is that CBA-IAMs poorly capture the potential changes in energy systems, lacking of modules that would integrate learning-by-doing and learning-by-using, supply chain optimisation and economies of scale, reduction of the perceived risks of the new technologies, etc. For this task, hybrid models can better represent high inertia derived from their representation of urban and transport system. Because reaching a single monetized estimate of the overall damages may be futile considering the diversity of impacts and the uncertainty about the future [34] (especially when there are no sign of convergence in energy use for developed countries with similar GDP per capita [33]), we should encourage the use of a wide range of models, notably simulation models, to explore the imaginable paths that could lead to a sustainable future.

To tackle nonequilibrium effects, learning effects, and bounded rationality, more heterodox approaches (*eg.* agend-based climate-economy models) may also be a precious resource [22, 36].

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